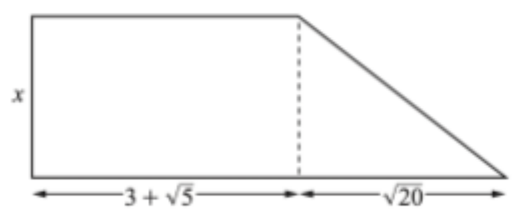
Chapter (3) Indices

0606/22/F/M/17

1. The diagram shows a trapezium made from a rectangle and a right-angled triangle. The dimensions, in centimetres, of the rectangle and triangle are shown. The area, in square centimetres, of the trapezium is $13 + 5\sqrt{5}$. **Without using a calculator,** find the value of x in the form $p + q\sqrt{5}$, where p and q are integers. [5]



$$\text{Area} = \frac{1}{2}(a+b) \times h$$

$$13 + 5\sqrt{5} = \frac{1}{2}(3+\sqrt{5}+\sqrt{20}+3+\sqrt{5}) \times x$$

$$26 + 10\sqrt{5} = (6+2\sqrt{5}+2\sqrt{5}) \times x$$

$$26 + 10\sqrt{5} = (6+4\sqrt{5})x$$

$$x = \frac{26+10\sqrt{5}}{6+4\sqrt{5}}$$

$$= \frac{13+5\sqrt{5}}{3+2\sqrt{5}} \times \frac{(3-2\sqrt{5})}{(3-2\sqrt{5})}$$

$$= \frac{39-26\sqrt{5}+15\sqrt{5}-10\times5}{9-4\times5}$$

$$= \frac{39-11\sqrt{5}-50}{9-20}$$

$$= \frac{-11-11\sqrt{5}}{-11}$$

$$= 1 + 1\sqrt{5}$$

2. (a) Express $(\sqrt[3]{-8x^9})(\sqrt[6]{x^{-3}})$ in the form ax^b, where a and b are constant to be found. [2]

$$(-8x^9)^{1/3}\ (x^{-3})^{1/6}$$

$$= (-2^3x^9)^{1/3}\ (x^{-3})^{1/6}$$

$$= (-2x^3)\ (x^{-1/2}) = -2x^{5/2}$$

$a = -2$

$b = \frac{5}{2}$

(b) Hence solve the equation $(\sqrt[3]{-8x^9})(\sqrt[6]{x^{-3}}) = -6250$. [2]

$$\cancel{-}2x^{5/2} = \cancel{-}6250$$

$$x^{5/2} = 3125$$

$$x = (3125)^{2/5}$$

$$x = 25$$

0606/11/M/J/17

3. Simplify $\sqrt{x^8y^{10}} \div \sqrt[3]{x^3y^{-6}}$, giving your answer in the form x^ay^b, where a and b are integers. [2]

$$(x^8y^{10})^{1/2} \div (x^3y^{-6})^{1/3}$$

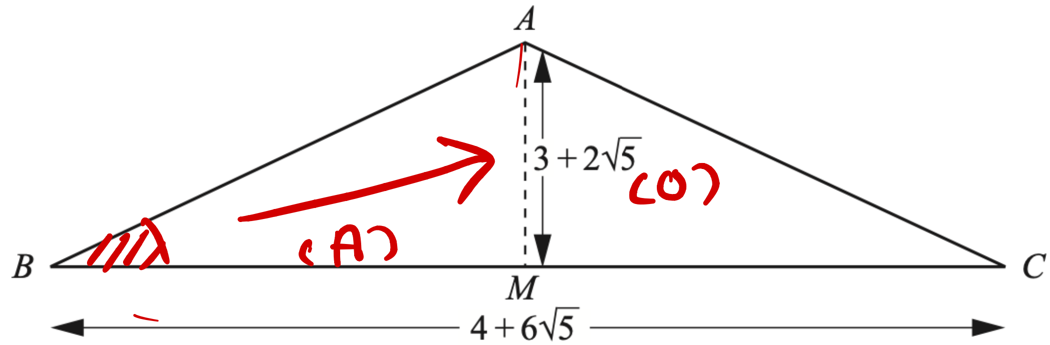
$$= x^4y^5 \div x^1y^{-2}$$

$$= x^3y^{5-(-2)}$$

$$= x^3y^7 \qquad a=3,\ b=7$$

0606/13/M/J/17

4. In this question, all dimensions are in centimetres.



The diagram show an isosceles triangle ABC, where $AB = AC$. The point M is the midpoint of BC. Given that $AM = 3 + 2\sqrt{5}$ and $BC = 4 + 6\sqrt{5}$, find, **without using a calculator,**

(i) the area of triangle ABC, [2]

Area $= \frac{1}{2}bh$

$= \frac{1}{2} \times (4+6\sqrt{5}) \times (3+2\sqrt{5})$

$= (2+3\sqrt{5}) \times (3+2\sqrt{5})$

$= 6+4\sqrt{5}+9\sqrt{5}+6\times 5$

$= 6+13\sqrt{5}+30$

$= 36+13\sqrt{5}$

(ii) tan ABC, giving your answer in the form $\frac{a+b\sqrt{5}}{c}$, where a, b and c are positive integers.

[3]

$\tan ABC = \frac{O}{A}$

$= \frac{3+2\sqrt{5}}{2+3\sqrt{5}} \times \frac{(2-3\sqrt{5})}{(2-3\sqrt{5})}$

$= \frac{6-9\sqrt{5}+4\sqrt{5}-6\times 5}{4-9\times 5}$

$= \frac{6-5\sqrt{5}-30}{4-45} = \frac{-24-5\sqrt{5}}{-41}$

$= \frac{24+5\sqrt{5}}{41}$

The Maths Society

0606/21/M/J/17

5. **Do not use a calculator in this question.**

(a) Show that $\sqrt{24} \times \sqrt{27} + \frac{9\sqrt{30}}{\sqrt{15}}$ can be written in the form $a\sqrt{2}$, where a is an integer. [3]

$$= 2\sqrt{6} \times 3\sqrt{3} + 9\sqrt{2}$$

$$\frac{\sqrt{30}}{\sqrt{15}} = \sqrt{\frac{30}{15}} = \sqrt{2}$$

$$= 6\sqrt{18} + 9\sqrt{2}$$

$$= 6 \times 3\sqrt{2} + 9\sqrt{2}$$

$$= 18\sqrt{2} + 9\sqrt{2} = 27\sqrt{2}$$

(b) Solve the equation $\sqrt{3}(1 + x) = 2(x - 3)$, giving your answer in the form $b + c\sqrt{3}$, where b and c are integers. [3]

$$\sqrt{3} + \sqrt{3}x = 2x - 6$$

$$\sqrt{3}x - 2x = -6 - \sqrt{3}$$

$$x(\sqrt{3} - 2) = -6 - \sqrt{3}$$

$$x = \frac{-6 - \sqrt{3}}{\sqrt{3} - 2} \times \frac{\sqrt{3} + 2}{\sqrt{3} + 2}$$

$$= \frac{-6\sqrt{3} - 12 - 3 - 2\sqrt{3}}{3 - 4}$$

$$= \frac{-8\sqrt{3} - 15}{-1}$$

$$= 8\sqrt{3} + 15$$

$$= 15 + 8\sqrt{3} \Leftarrow$$

0606/22/M/J/17

6. **Without using a calculator**, express $\left(\frac{1+\sqrt{5}}{3-\sqrt{5}}\right)^{-2}$ in the form $a + b\sqrt{5}$, where a and b are integers. [5]

$$\left(\frac{3-\sqrt{5}}{1+\sqrt{5}}\right)^2$$

$$= \frac{9-6\sqrt{5}+5}{1+2\sqrt{5}+5}$$

$$= \frac{14-6\sqrt{5}}{6+2\sqrt{5}}$$

$$= \frac{7-3\sqrt{5}}{3+\sqrt{5}} \frac{\times(3-\sqrt{5})}{\times(3-\sqrt{5})}$$

$$= \frac{21-7\sqrt{5}-9\sqrt{5}+3\times 5}{9-5} = \frac{21-16\sqrt{5}+15}{4} = \frac{36-16\sqrt{5}}{4}$$

$$= 9-4\sqrt{5} \leftarrow$$

0606/23/M/J/17

7. Express $\frac{(5\sqrt{q})^3}{(625p^{12}q)^{-\frac{1}{4}}}$ in the form $5^a p^b q^c$, where a, b and c are constants. [3]

$$\frac{5^3 q^{3/2}}{(5^4 p^{12} q)^{1/4}} = \frac{5^3 q^{3/2}}{5^1 p^3 q^{1/4}}$$

$$= 5^2 p^{-3} q^{3/2 - \frac{1}{4}}$$

$$= 5^2 p^{-3} q^{5/4}$$

$a = 2$

$b = -3$

$c = \frac{5}{4}$